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A MULTI-PRODUCT EOQ MODEL FOR EXPONENTIAL INCREASING DEMAND AND WEIBULL DISTRIBUTION DETERIORATION WITH BUDGET CONSTRAINTS CONSIDERATION AND SHORTAGES

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ABSTRACT

This paper consider a multi-product EOQ model with fuzzy for exponential increasing demand and Weibull distribution deterioration with Budget constraints and Shortages by integrating the product pricing and order sizing decisions. Minimize the total cost and illustrate with an example using LINGO.

Key-words: *Multi-product model, Weibull distribution deterioration, exponential increasing demand, fuzzy, α-cut andGlobal criteria method.*

INTRODUCTION

The inventory models are always formulated and solve to determine the optimal stock of commodities to meet the future demand which may either arise at a constant rate or may vary with time. When the items of commodity are kept in the stock as an inventory for fulfilling the demand, there may be the deterioration of items in the inventory system which may occur due to one or many factors viz storage condition, weather condition etc. Some items in the inventory system may deteriorate where as others can be stored for an indefinite period without deterioration. The deterioration is usually a function of the total amount of inventory on hand. Deterioration is defined as decay, damage or spoilage. Food items, photographic films, drugs, pharmaceutical, chemicals, electronics compounds and radioactive substance are some example of items in which sufficient deterioration may occur during the normal storage period of the units and consequently this loss must be taken into account while analyzing the inventory system. Inventory models in which unit deteriorate while in storage, have drawn attention of various researchers in recent years. Decay in radioactive elements, spoilage in food grain storage, pilferages from on hand inventory etc. are continuous in time and roughly proportionally to the on hand inventory.

The classical EOQ model is based on the assumption that the demand rate for the product is constant and that it is independent of the price of the product. This is not true in many situations, in the case of a monopolistic firm. A monopolist always influences the demand of the product by manipulating its price. In such cases the demand rate should be treated as a decision variable. Moreover, the order size of a product should normally depend on its price within (Whitin, 1955) formulated an EOQ model taking the demand to be a negatively sloped function of the price. However, he did not solve the model explicitly. Porteus (1485) discussed the issue of setup reduction in an EOQ model in which the demand rate was taken as a decision variable depending on the price. Cheng (1990) presented a multi-product EOQ model that integrated pricing and order sizing decisions to maximize profit with storage space and inventory investment constraints. By removing some shortcomings of the model, it was further improved by Chen and Min (1994). Later, different part of it's also improved by Intrilligator (1971), Covert *et al*., (1973), and Philip (1974). Agarwal and Jain (1997). In this all models, deterioration and Shortages in inventory are not taken into account. These being realistic factors associated with the inventory of any item, should be taken into account to make the model more realistic. Agarwal and Jain (1997) consider shortages. But in Chakraborty *et al*., (1998), Dev and Chaudhuri, (1987), Dave, (1989), Goswami and Chaudhuri, (1991) and Jalan *et al*., (1996), Kundu and Chakraborty (2009) they consider deterioration and Shortages.

In this paper, we consider a multi-product EOQ model that—i) integrating the product pricing and order sizing decisions, ii) allow shortages in inventory and iii) deals with two parameters Weibully distribution deterioration items. Demand is assumed to be having one parameter exponential distribution. Our object is to minimize the total cost. We take into consideration two practical constraints, namely, storages space and inventory investment limitation. This paper is a extension of Jalan and Chaudhuri, (2000). Chen and Min (1994) considered limitation on the total inventory carrying cost to be the limitation on inventory investment. We do not appreciate this concept limitation of inventory investment. In our model, the inventory investment is the money required for procuring the inventory by purchase or production as the case may be. Our problem is formulated as a constrained non-linear optimizing problem which can be solved by applying the Khun-Tucher optimality conditions.

Notation and assumption: We consider here a multi-product EOQ model which consists of product of different types and there are restrictions on the maximum amount of capital invested in stock at any time and also on the maximum storages space available in the warehouse.

The notations for the model are as follows:
n=total number of types of product produced by the

firm, R_i=demand rate for the product i, q_i=order size the product i, s_i=shortage size of the product i, $Q_i(t)$=inventory of the i^{th} item at any time t, C_{11}=holding cost per unit time for product i, C_{12}=shortage cost per unit time for product i, C_{13}=setup cost for product i per production run, R_i=the unit cost of production of product i, T=total cycle time, f_i=storage space requirement per unit of product i, P_i=unit selling price of product I, F=total fixed cost of production and administration, A=total storage space available, M=the amount of capital to be invested on inventory, θ_i=fraction of the on hand inventory of the product I which deteriorates per unit time, $0<\theta<1$, C(q, s, t)=total cost of the products.

The following basic assumptions about the model are:

1. All products have an equal replenishment cycle of length T. This facilitates operational convenience. This practice is usually followed in multi-product inventory models. If a firm dealing with (purchase or production) several types of products, initiates procurement actions for different products at different times, the situation will be operationally unchangeable and setup(or ordering) cost will have to be incurred each time as a procurement action is initiated.
2. Replenishment of the products is instantaneous with zero time.
3. Shortages in inventory of each product are allowed.
4. The demand rates are increases exponentially and for i^{th} item is given by the function $R_i=ae^{bt}$, $a>0$, $0<b<1$.
5. The rate of deterioration at any time t>0 follow the two-parameter weibull distribution as $\theta_i=\alpha\beta t^{(\beta-1)}$, where α $(0<\alpha<1)$ is the scale parameter and $\beta(>0)$ is the shape parameter.
6. A deteriorating item neither repaired nor replaced during the cycle.

The Model: The amount of stock for the i^{th} item is q_i (i=1, 2... n) at the time t=0. In the interval [0, T], the inventory level gradually decreases mainly to meet demands and partly for deterioration. By this process, the inventory level reaches zero at time t_1 (<T) and then shortages are allowed to occur in the interval $[t_1, T]$. The cycle then repeated itself. The amount of shortages for the i^{th} item is s_i (i=1, 2... n) at time t=T. The differential equations for the instantaneous inventory $Q_i(t)$ of the i^{th} item at any time t in [0, T] for exponential demand and weibull distribution deterioration, are given by

$$\frac{dQ_i(t)}{dt} + \alpha\beta t^{(\beta-1)} Q_i(t) = -ae^{bt}, \quad 0 \le t \le t_1 \quad (1)$$

$$\frac{dQ_i(t)}{dt} = -ae^{bt}, \quad t_1 \le t \le T \quad (2)$$

The boundary conditions are:

$Q_i(0)=q_i$, $Q_i(t_1)=0$ and $Q_i(T)=-s_i$ (3)

The solutions of the differential equation (1) and (2) are given by

$$Q_i(t) = \{q_i - a(t + \frac{b}{2}t^2 + \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{\alpha b t_1^{\beta+2}}{\beta+2})\} e^{-at^{\beta}}, \quad 0 \le t \le t_1 \quad (4)$$

And $$Q_i(t) = \frac{a}{b}\{e^{bt_1} - e^{bt}\}, \quad t_1 \le t \le T \quad (5)$$

At $t=t_1$ and using boundary conditions, we get

$$q_i = a(t_1 + \frac{b}{2}t_1^2 + \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{\alpha b t_1^{\beta+2}}{\beta+2}) \quad (6)$$

This gives the value of t_1 where the shortage starts.

Now, The total inventory of the i^{th} item in $[0, t_1]$ is

$$H_i = \int_0^{t1} Qi(t)dt = q_i(t_1 - \frac{\alpha t_1^{\beta+1}}{\beta+1}) - a(\frac{t_1^2}{2} + \frac{bt_1^3}{6} - \frac{\alpha\beta t^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha\beta b t^{\beta+3}}{2(\beta+2)(\beta+3)}) \quad (7)$$

Total deterioration of the i^{th} item in $[0, t_1]$ is

$$F_i = \int_0^{t_1} \theta_i Q_i(t)dt$$

$$= q_i \alpha t_1^{\beta} - a(\frac{\alpha\beta t_1^{\beta+1}}{\beta+1} + \frac{\alpha\beta b t_1^{\beta+2}}{\beta+2}) \quad (8)$$

Total shortages of the i^{th} item in $[0, t_1]$ is

$$D_i = \int_{t_1}^{T} \{-Q_i(t)\}dt = \frac{s_i^2 e^{-bt_1}}{2a} \quad (9)$$

The total cost for n-items is C (q, s, T) = (Setup cost + Production cost + Inventory holding cost + Deterioration cost + Shortage cost) for n-items + Fixed cost

$$= \sum_{i=1}^{n} [C_{3i} + \{r_i + \alpha r_i t_1^{\beta} + C_{1i}(t_1 - \frac{\alpha t_1^{\beta+1}}{\beta+1})\} q_i - aC_{1i}(\frac{t_1^2}{2} + \frac{bt_1^3}{6} - \frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha\beta b t_1^{\beta+3}}{2(\beta+2)(\beta+3)})$$
$$- a\alpha\beta(\frac{t_1^{\beta+1}}{\beta+1} + \frac{bt_1^{\beta+2}}{2(\beta+2)}) r_i + \frac{C_{2i} s_i^2 e^{-bt_1}}{2a}] + FT$$

Total cost per unit cycle is C (q, s, T)

$$=\frac{1}{T}\sum_{i=1}^{n}[C_{3i}+\{r_i+\alpha r_i t_1^{\beta}+C_{1i}(t_1-\frac{\alpha t_1^{\beta+1}}{\beta+1})\}q_i-aC_{1i}(\frac{t_1^2}{2}+\frac{bt_1^3}{6}-\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha\beta b t_1^{\beta+3}}{2(\beta+2)(\beta+3)})$$
$$-a\alpha\beta(\frac{t_1^{\beta+1}}{\beta+1}+\frac{bt_1^{\beta+2}}{2(\beta+2)})r_i+\frac{C_{2i}s_i^2e^{-bt_1}}{2a}]+F \quad ....(10)$$

Now, our object is to minimize the total cost subject to the storage and inventory investment constraints. The problem is as Minimize C (q, s, T)

$$=\frac{1}{T}\sum_{i=1}^{n}[C_{3i}+\{r_i+\alpha r_i t_1^{\beta}+C_{1i}(t_1-\frac{\alpha t_1^{\beta+1}}{\beta+1})\}q_i-aC_{1i}(\frac{t_1^2}{2}+\frac{bt_1^3}{6}-\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha\beta b t_1^{\beta+3}}{2(\beta+2)(\beta+3)})$$
$$-a\alpha\beta(\frac{t_1^{\beta+1}}{\beta+1}+\frac{bt_1^{\beta+2}}{2(\beta+2)})r_i+\frac{C_{2i}s_i^2e^{-bt_1}}{2a}]+F \quad (11$$

Subject to $\sum_{i=1}^{n} f_i q_i \le A$ (12)

And $\sum_{i=1}^{n} r_i q_i \le M$ (13)

Where $q_i \ge 0$, $s_i \ge 0$, $T \ge 0$. Again, our object is to maximize the total cost subject to the storage and inventory investment Constraints. The problem is as Maximize C (q, s, T)

$$=\frac{1}{T}\sum_{i=1}^{n}[C_{3i}+\{r_i+\alpha r_i t_1^{\beta}+C_{1i}(t_1-\frac{\alpha t_1^{\beta+1}}{\beta+1})\}q_i-aC_{1i}(\frac{t_1^2}{2}+\frac{bt_1^3}{6}-\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha\beta b t_1^{\beta+3}}{2(\beta+2)(\beta+3)})$$
$$-a\alpha\beta(\frac{t_1^{\beta+1}}{\beta+1}+\frac{bt_1^{\beta+2}}{2(\beta+2)})r_i+\frac{C_{2i}s_i^2e^{-bt_1}}{2a}]+F \quad (14)$$

Subject to $\sum_{i=1}^{n} f_i q_i \le A$ (15) And $\sum_{i=1}^{n} r_i q_i \le M$ (16)

Where $q_i \ge 0$, $s_i \ge 0$, $T \ge 0$, The numerical example is discussed later after its fuzzy model.

Fuzzy: Real world problems are generally associated with different types of uncertainties and imprecision's. In the past, a considerable amount of effort was made to model those uncertainties and imprecision's. Prior to 1965, people used to consider probability theory as the prime agent for dealing with uncertainties. It is to be noted that the above logic uses the concept of classical or crisp set theory. Prof. Zadeh argued that the probability theory can handle only one out of several different types of possible uncertainties. Thus, there are some uncertainties, which cannot be tackled using the probability theory. So, to handle this we have to require a new concept, which is known as fuzzy. It has two parts, element is one and the membership function of the element is the other. In real life, we cannot predict to determine the exact demand. We can consider only the certain approximation of the demand. So the demand to be consider as fuzzy.

Let, the demand and deterioration are as $R_i = ae^{bt}$, $a>0$, $0<\theta<1$ And $\theta_i = \alpha\beta t^{\beta-1}$, $0<\alpha<1$, $\beta>0$, We consider 'a' in demand as a triangular fuzzy number, and defined as $\tilde{a}= (a_1, a_2, a_3)$. Then the corresponding membership function is defined as

$\mu_{\tilde{a}}(x)=\frac{x-a_1}{a_2-a_1}$, $a_1 \le x \le a_2$

$=\frac{a_3-x}{a_3-a_2}$, $a_2 \le x \le a_3$ =0 , otherwise

α-cut of a fuzzy set A(x): It is a set consisting of elements x of the universal set X, whose membership values are either greater than or equal to the value of α. It is denoted by the symbol $^{\alpha}\mu_A(x)$ and is defined as $^{\alpha}\mu_A(x)$ = {x / $\mu_A(x) \ge \alpha$}. Taking α-cut of $\tilde{a}$, Due to over lacking of α, we consider here η-cut as α-cut and we have $\mu_{\tilde{a}}(x) \ge \eta$, $0 \le \eta \le 1$. Then, $\frac{x-a_1}{a_2-a_1} \ge \eta$ and $\frac{a_3-x}{a_3-a_2} \ge \eta$ i.e. $x \ge a_1+\eta(a_2-a_1)$ and $x \le a_3-\eta(a_3-a_2)$.

So an η-cut of $\tilde{a}$ can be expressed by then following interval

$\tilde{a}(\eta)=[a_1+\eta(a_2-a_1), a_3-\eta(a_3-a_2)]$,

$\eta \in [0,1]$ where $a^-(\eta)$ =$a_1+\eta(a_2-a_1)$ and

$a^{+}(\eta)$ =a3-η(a3-a2) are known as lower cut and upper cut respectively.

Fuzzy Model: The system of equation for the given problem in [1, 0] as

$$\frac{dQ_i(t)}{dt} + \alpha\beta t^{(\beta-1)} Q_i(t) = -\tilde{a}(\eta)e^{bt},\ 0 \le t \le t_1 \quad (17)$$

$$\frac{dQ_i(t)}{dt} = -a(\eta)e^{bt},\ t_1 \le t \le T \quad (18)$$

With the boundary conditions are: $Q_i(0)=q_i$, $Q_i(t_1)=0$ and $Q_i(T)=-s_i$. Now using **η-cut** the system of equation (14&15) can be rewritten as

$$\frac{dQ^{+}_i(t)}{dt} + \alpha\beta t^{(\beta-1)} Q_i(t) = -a^{-}(\eta)e^{bt},\ 0 \le t \le t_1 \quad (19)$$

$$\frac{dQ^{-}_i(t)}{dt} + \alpha\beta t^{(\beta-1)} Q_i(t) = -a^{+}(\eta)e^{bt},\ 0 \le t \le t_1 \quad (20)$$

And $$\frac{dQ^{+}_i(t)}{dt} = -a^{-}(\eta)e^{bt},\ t_1 \le t \le T \quad (21)$$

$$\frac{dQ^{-}_i(t)}{dt} = -a^{+}(\eta)e^{bt},\ t_1 \le t \le T \quad (22)$$

Where, $a^{-}(\eta)=a_1+\eta(a_2-a_1)$ and $a^{+}(\eta)=a_3-\eta(a_3-a_2)$ are the lower cut and upper cut respectively.

The solutions of the differential equation (16) and (17) are given by

$$Q_i^{+}(t) = \{q_i - a^{-}(\eta)\left(t + \frac{b}{2}t^2 + \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{\alpha b t_1^{\beta+2}}{\beta+2}\right)\}\, e^{-a t^{\beta}},\ 0\le t\le t_1 \quad (23)$$

$$Q_i^{-}(t) = \{q_i - a^{+}(\eta)\left(t + \frac{b}{2}t^2 + \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{\alpha b t_1^{\beta+2}}{\beta+2}\right)\}\, e^{-a t^{\beta}},\ 0\le t\le t_1 \quad (24)$$

And $$Q_i^{+}(t) = \frac{a^{-}(\eta)}{b}\{e^{bt_1} - e^{bt}\},\quad t_1\le t\le T \quad (25)$$

$$Q_i^{-}(t) = \frac{a^{+}(\eta)}{b}\{e^{bt_1} - e^{bt}\},\ t_1\le t\le T \quad (26)$$

Where, $a^{-}(\eta)=a_1+\eta(a_2-a_1)$ and $a^{+}(\eta)=a_3-\eta(a_3-a_2)$

At $t=t_1$ and using boundary conditions, from (20) & (21), we get

$$q_i = a^{-}(\eta)\left(t_1 + \frac{b}{2}t_1^2 + \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{\alpha b t_1^{\beta+2}}{\beta+2}\right) \quad (27)$$

$$q_i = a^{+}(\eta)\left(t_1 + \frac{b}{2}t_1^2 + \frac{\alpha t_1^{\beta+1}}{\beta+1} + \frac{\alpha b t_1^{\beta+2}}{\beta+2}\right) \quad (28)$$

Where, $a^{-}(\eta)=a_1+\eta(a_2-a_1)$ and $a^{+}(\eta)=a_3-\eta(a_3-a_2)$

NOW, The total upper inventory of the i^{th} item in $[0, t_1]$ is

$$H_i^{+} = \int_0^{t1} Q_i^{+}(t)dt = q_i\left(t_1 - \frac{\alpha t_1^{\beta+1}}{\beta+1}\right) - a^{-}(\eta)\left(\frac{t_1^2}{2} + \frac{bt_1^3}{6} - \frac{\alpha\beta t^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha\beta b t^{\beta+3}}{2(\beta+2)(\beta+3)}\right) \quad (29)$$

Similarly, The total lower inventory of the i^{th} item in $[0, t_1]$ is

$$H_i^{-} = \int_0^{t1} Q_i^{-}(t)dt = q_i\left(t_1 - \frac{\alpha t_1^{\beta+1}}{\beta+1}\right) - a^{+}(\eta)\left(\frac{t_1^2}{2} + \frac{bt_1^3}{6} - \frac{\alpha\beta t^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha\beta b t^{\beta+3}}{2(\beta+2)(\beta+3)}\right) \quad (30)$$

Where, $a^{-}(\eta)=a_1+\eta(a_2-a_1)$ and $a^{+}(\eta)=a_3-\eta(a_3-a_2)$ Total upper deterioration of the i^{th} item in $[0, t_1]$ is

$$F_i^{+} = \int_0^{t_1} \theta_i Q_i^{+}(t)dt = q_i \alpha t_1^{\beta} - a^{-}(\eta)\left(\frac{\alpha\beta t_1^{\beta+1}}{\beta+1} + \frac{\alpha\beta b t_1^{\beta+2}}{\beta+2}\right) \quad (31)$$

Total lower deterioration of the i^{th} item in $[0, t_1]$ is

$$F_i^{-} = \int_0^{t_1} \theta_i Q_i^{-}(t)dt = q_i \alpha t_1^{\beta} - a^{+}(\eta)\left(\frac{\alpha\beta t_1^{\beta+1}}{\beta+1} + \frac{\alpha\beta b t_1^{\beta+2}}{\beta+2}\right) \quad (32)$$

Where, $a^{-}(\eta)=a_1+\eta(a_2-a_1)$ and $a^{+}(\eta)=a_3-\eta(a_3-a_2)$ Total upper shortages of the i^{th} item in $[t_1, T]$ is

$$D_i^{+} = \int_{t_1}^{T} \{-Q_i^{+}(t)\}dt = \frac{s_i^2 e^{-bt_1}}{2a^{-}(\eta)} \quad (33)$$

Total lower shortages of the i^{th} item in $[t_1, T]$ is $D_i^{-} = \int_{t_1}^{T} \{-Q_i^{-}(t)\}dt = \frac{s_i^2 e^{-bt_1}}{2a^{+}(\eta)}$ (34)

Where, $a^{-}(\eta)=a_1+\eta(a_2-a_1)$ and $a^{+}(\eta)=a_3-\eta(a_3-a_2)$

The total upper cost for n-items is C(q,s,T) = (Setup cost + Production cost + Upper inventory holding cost+ Upper deterioration cost + Upper shortage cost) for n-items + Fixed cost

$$=\sum_{i=1}^{n}[C_{3i}+\{r_i+\alpha r_i t_1^{\beta}+C_{1i}(t_1-\frac{\alpha t_1^{\beta+1}}{\beta+1})\}q_i-a^{-}(\eta)C_{1i}(\frac{t_1^{2}}{2}+\frac{bt_1^{3}}{6}-\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha\beta b t_1^{\beta+3}}{2(\beta+2)(\beta+3)})$$
$$-a^{-}(\eta)\alpha\beta(\frac{t_1^{\beta+1}}{\beta+1}+\frac{bt_1^{\beta+2}}{2(\beta+2)})r_i+\frac{C_{2i}s_i^{2}e^{-bt_1}}{2a}]+FT$$

Where, $a^{-}(\eta)$ =a_1+η (a_2-a_1) and $a^{+}(\eta)$ =a_3-η (a_3-a_2). The total lower cost for n-items is C (q,s,T) = (Setup cost + Production cost + lower inventory holding cost + Lower deterioration cost + lower shortage cost) for n-items + Fixed cost

$$=\sum_{i=1}^{n}[C_{3i}+\{r_i+\alpha r_i t_1^{\beta}+C_{1i}(t_1-\frac{\alpha t_1^{\beta+1}}{\beta+1})\}q_i-a^{+}(\eta)C_{1i}(\frac{t_1^{2}}{2}+\frac{bt_1^{3}}{6}-\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha\beta b t_1^{\beta+3}}{2(\beta+2)(\beta+3)})$$
$$-a^{+}(\eta)\alpha\beta(\frac{t_1^{\beta+1}}{\beta+1}+\frac{bt_1^{\beta+2}}{2(\beta+2)})r_i+\frac{C_{2i}s_i^{2}e^{-bt_1}}{2a}]+FT$$

Where, $a^{-}(\eta)$ =a_1+η (a_2-a_1) and $a^{+}(\eta)$ =a_3-η (a_3-a_2). Now, our object is to minimize the total upper cost subject to the storage and inventory investment constraints. Then the problem is as Minimize C (q, s, T)

$$=\frac{1}{T}\sum_{i=1}^{n}[C_{3i}+\{r_i+\alpha r_i t_1^{\beta}+C_{1i}(t_1-\frac{\alpha t_1^{\beta+1}}{\beta+1})\}q_i-a^{-}(\eta)C_{1i}(\frac{t_1^{2}}{2}+\frac{bt_1^{3}}{6}-\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha\beta b t_1^{\beta+3}}{2(\beta+2)(\beta+3)})$$
$$-a^{-}(\mu)\alpha\beta(\frac{t_1^{\beta+1}}{\beta+1}+\frac{bt_1^{\beta+2}}{2(\beta+2)})r_i+\frac{C_{2i}s_i^{2}e^{-bt_1}}{2a}]+F \quad ....(35)$$

Subject to $\sum_{i=1}^{n} f_i q_i \le A$ (36)

And $\sum_{i=1}^{n} r_i q_i \le M$ (37)

Where $q_i \ge 0$, $s_i \ge 0$, T≥0 and $a^{-}(\eta)$ =a_1+η (a_2-a_1)

Similarly, our object is to minimize the total lower cost subject to the storage and inventory investment constraints. The problem is as **Minimize (q, s, T)**

$$=\frac{1}{T}\sum_{i=1}^{n}[C_{3i}+\{r_i+\alpha r_i t_1^{\beta}+C_{1i}(t_1-\frac{\alpha t_1^{\beta+1}}{\beta+1})\}q_i-a^{+}(\eta)C_{1i}(\frac{t_1^{2}}{2}+\frac{bt_1^{3}}{6}-\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha\beta b t_1^{\beta+3}}{2(\beta+2)(\beta+3)})$$
$$-a^{+}(\mu)\alpha\beta(\frac{t_1^{\beta+1}}{\beta+1}+\frac{bt_1^{\beta+2}}{2(\beta+2)})r_i+\frac{C_{2i}s_i^{2}e^{-bt_1}}{2a}]+F \quad ....(38)$$

Subject to $\sum_{i=1}^{n} f_i q_i \le A$ (39)

And $\sum_{i=1}^{n} r_i q_i \le M$ (40)

Where $q_i \ge 0$, $s_i \ge 0$, T≥0 and $a^{+}(\eta)$ =a_3-η (a_3-a_2)

Numerical Example:

CASE—1: We consider the values corresponding to the variables to the problem mentioned above are as follows: Let, f_1=$24, f_2=$28, α=0.006, β=1.5, a=100, b=0.03, A=$1400, M=$1000, C_{11}=$5, C_{12}=$4.5, C_{21}=$13, C_{22}=$14.7, C_{31}=$100, C_{32}=$100, T=1, r_1=$18, r_2=$20, S_1=18, S_2=20; F=$50; Then the optimum minimized cost is Min.C (q, s, T) =$1091.789. And for a_1=50, a_2=100, a_3=150, η=0.3, $a^{-}(\eta)$ =65, $a^{+}(\eta)$ =135. Then the optimum minimized upper cost is Min.C (q, s, T) =$1099.308. Then the optimum minimized lower cost is Min.C (q, s, T) =$1079.924

CASE—2: Again let, f_1=$20, f_2=$22, α=0.006, β=0.4, a=100, b=0.03, A=$1600, M=$1200, C_{11}=$5, C_{12}=$5.2, C_{21}=$14, C_{22}=$15, C_{31}=$100, C_{32}=$100, T=1, r_1=$15, r_2=$12, S_1=3, S_2=6; F=$50; Then the optimum maximize cost is Max.C (q, s, T) =$1126.347. And for a_1=50, a_2=100, a_3=150, η=0.3, $a^{-}(\eta)$ =70, $a^{+}(\eta)$ =130. Then the optimum maximize upper cost is Max.C (q, s, T) =$1128.890. Then the optimum maximized lower cost is Max.C (q, s, T) =$1120.947.**Finally**, we optimize the cost function using the method of Global Criteria method, defined as:

Global Criteria Method: The model represented by (11), (12) & (13) is a multi-objective model which is solved by Global Criteria (GC) Method with the help of Generalized Reduction Gradient technique. The Multi-Objective Non Linear Integer Programming (MONLIP) problems are solved by Global Criteria Method converting it into a single objective optimization problem. The solution procedure is as follows:

Step-1: Solve the multi-objective programming problem by (11), (12) & (13) as a single objective problem using one objective at a time ignoring the other.

Step-2: From the result of Step-1, determine the ideal objective vector, (say $TC^{+\min}, TC^{-\min}$) and (say $TC^{+\max}, TC^{-\max}$). Here the ideal objective vector is use as a reference point. The problem is then to solve the auxiliary problem:

Min (GC) =Minimize

$$\{(\frac{TC^{+} - TC^{+\max}}{TC^{+\max} - TC^{+\min}})^{p} + (\frac{TC^{-} - TC^{-\min}}{TC^{-\max} - TC^{-\min}})^{p}\}^{\frac{1}{p}}$$

(41)

Where $1 \le p < \infty$. As usual value of p is 2. The method is also sometimes called compromise Programming. Since,

TC^{+}=\$1126.347;

$TC^{+\max} = \$1128.890; TC^{-\max} = \$1120.947$;

TC^{-}=\$1089.633;

$TC^{+\min} = \$1099.308; TC^{-\min} = \$1076.861$;

Minimize (GC) =\$0.9597188.

CONCLUSION

The model presented above is distinctly a significant improvement for exponential increasing demand and weibull distribution deterioration on the following accounts: It takes into account the fact that physical goods kept in stock undergo decay or deterioration over time. It corporate occurrence of shortages in inventory into the model. It is an established fact in the inventory literature that the average system cost can be significantly reduced by permitting shortages in inventory. The third point of departure lies in the constraint on inventory investment. They imposed a limitation on the amount of the total inventory carrying cost. They could not provide always numerical example due to non-availability of a suitable software package, but we provide a numerical example using Global criteria method and LINGO.

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